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ON AFFLECK-DINE-SEIBERG SUPERPOTENTIAL AND MAGNETIC MONOPOLES IN SUPERSYMMETRIC QCD

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ABSTRACT

Certain exact results in supersymmetric gauge theories are generated by non-perturbative effects different from instantons. In supersymmetric QCD with N colours and N_F fundamental flavours we examine the Affleck-Dine-Seiberg (ADS) superpotential using controlled semi-classical analysis. We show how for $N_F < N - 1$ the ADS superpotential arises from monopole contributions to the path integral of the supersymmetric gauge theory compactified on $\mathbb{R}^3 \times S^1$. These are the monopole effects leading to gaugino condensation and confinement of the low-energy $SU(N - N_F)$ supersymmetric gauge theory.

I Introduction and Motivation

One of the famous exact results in supersymmetry is the Affleck-Dine-Seiberg (ADS) superpotential [1] generated in supersymmetric QCD with N_F fundamental flavours and gauge group $SU(N)$

$$\mathcal{W}_{\text{ADS}}^{N_F, N} = (N - N_F) \left(\Lambda_{N_F, N}^{3N - N_F} \det_{N_F}(\tilde{Q}^r Q_s) \right)^{1N - N_F}. \quad (1.1)$$

It is well known that for $N_F < N - 1$ the ADS superpotential receives no contributions from instantons, and arises from non-perturbative effects associated with gaugino condensation in the unbroken $SU(N - N_F)$ gauge group [1]. In our earlier work [2] we showed that gaugino condensation is generated by monopole contributions to the path integral of supersymmetric gauge theories compactified on $\mathbb{R}^3 \times S^1$. In this paper we make an obvious link between these two ideas [1, 2] and explicitly derive the ADS superpotential in the background of $N - N_F$ varieties of $SU(N - N_F)$ monopoles embedded in the $SU(N)$ supersymmetric QCD with N_F flavours. We will also explain why this semi-classical derivation is valid in the a priori strongly coupled theory. For related work on $\mathbb{R}^3 \times S^1$ in a different context see [3–5].

The last few years have seen an impressive agreement between known exact results in supersymmetric gauge theories and direct field-theoretical calculations. In most of the cases considered, the non-perturbative information contained in the exact results [6–8] was restricted to multi-instanton contributions [9–11]. At the same time, some of the known exact results in supersymmetric gauge theories are generated by non-perturbative effects different from instantons. We expect that these non-perturbative effects can still be evaluated and understood semi-classically, but in slightly unusual settings. The motivation of this letter and of its predecessor Ref. [2] is to show how this works for two important examples: gaugino condensation and the ADS superpotential. In both cases there is a non-Abelian, asymptotically free (sub)group which makes the theory strongly coupled. This is an obstacle for a direct application of semi-classical analysis, which makes these cases qualitatively different from the weakly coupled scenarios:

1. In the Seiberg-Witten case [6] the $SU(N)$ gauge group is broken by the Higgs mechanism to the Abelian subgroup $U(1)^{N-1}$ with effective couplings determined by the VEVs. The holomorphic nature of this dependence, guaranteed by non-renormalization theorems in $\mathcal{N} = 2$, then allows one to travel smoothly to an arbitrarily weak coupling regime saturated by the relevant semi-classical physics: perturbation theory, and multi-instanton effects calculated in [9, 10].
2. In general, when there is no unbroken non-Abelian subgroup, the F-terms in any $\mathcal{N} = 1$ supersymmetric theory can be usually determined with a constrained instanton calcula-

tion [1, 15–18] as reviewed in [19]. A simple example of this set-up is the supersymmetric QCD with $N_F = N - 1$.

3. In the AdS/CFT programme [7] the non-Abelian gauge group is unbroken, but the β -function is zero, and for successful multi-instanton calculations [8, 11–14], the constant coupling can be taken as small as needed.

It has been suspected for a long time [20] that in the strongly coupled theories, instantons should be thought of as composite states of more basic configurations, the ‘instanton partons’. These partons would give important contributions to the non-perturbative dynamics at strong coupling. In Ref. [2] we identified the instanton partons with the N varieties of monopoles which appear when Minkowski space-time of $SU(N)$ supersymmetric gauge theory is partially compactified on $\mathbb{R}^3 \times S^1$. This compactification is required for two reasons. First, because of the finite radius β of S^1 , the monopole solutions have finite action, which makes them important semi-classically. Second, the gauge fields with Lorentz index referring to the S^1 direction develop vacuum expectation values [2]

$$\langle A_4 \rangle = \text{diag} \left(N - 1N\pi i\beta, N - 3N\pi i\beta, \dots, -N - 1N\pi i\beta \right), \quad (1.2)$$

which break the non-Abelian $SU(N)$ to the Abelian subgroup $U(1)^{N-1}$. As the VEVs are inversely proportional to the radius β , the theory becomes weakly coupled at small β and can be analysed semi-classically. To return to the strongly coupled theory in Minkowski space, we need to consider the opposite limit of large β . Since all the F-terms are holomorphic functions of the fields and since the VEVs of the fields (1.2) are holomorphic functions of β , the power of holomorphy [21] allows to analytically continue the semi-classical values of the F-terms to the strong-coupling regime.

In supersymmetric QCD with N colours and N_F fundamental flavours in the Higgs phase, the gauge group is broken by the matter VEVs to $SU(N - N_F)$. In Section II we will review the monopole calculus in the theory with $N_F = 0$ on $\mathbb{R}^3 \times S^1$. The monopole effects in this theory generate the gaugino condensate and give a mass to the dual (magnetic) photon which implies confinement of electric charges. In Section III we will include the matter fields and start in uncompactified Minkowski space. We will first integrate out the classically massive (Higgsed) fields. The resulting effective lagrangian will contain the massless mesons $M = \tilde{Q}Q$ and the strongly-interacting massless $SU(N - N_F)$ gauge supermultiplet, $W_{SU(N-N_F)}$. These two sectors will be coupled to each other in the effective lagrangian via non-renormalizable interactions generated perturbatively by integrating out massive fields propagating in the loops. The next step will be to integrate out the gauge supermultiplet, $W_{SU(N-N_F)}$. This is achieved in the monopole background in the $SU(N - N_F)$ theory compactified on $\mathbb{R}^3 \times S^1$. The superpotential for mesons – the ADS superpotential (1.1) – arises from the gauge-meson

interactions when the gauge supermultiplet is integrated out. Hence, the ADS superpotential (1.1) is a combination of perturbative and monopole effects. The analysis of Section III can be schematically represented as

$$\begin{aligned} & \int \mathcal{D}W_{SU(N)} \mathcal{D}Q \mathcal{D}\tilde{Q} \, e^{i \int d^4x \mathcal{L}(W_{SU(N)}, Q, \tilde{Q})} \\ &= \int \mathcal{D}W_{SU(N-N_F)} \mathcal{D}M \, e^{i \int d^4x \mathcal{L}_{\text{eff}}^{(1)}(W_{SU(N-N_F)}, M)} = \int \mathcal{D}M \, e^{i \int d^4x \mathcal{L}_{\text{eff}}^{(2)}(M)} . \end{aligned} \quad (1.3)$$

We stress that Eq. (1.3) represents a *direct* evaluation of the ADS superpotential contained in $\mathcal{L}_{\text{eff}}^{(2)}(M)$. In particular we are not appealing to renormalization group arguments relating expressions for different values of N_F . None of the meson degrees of freedom were integrated out on the right hand side of Eq. (1.3), and the F-term of $\mathcal{L}_{\text{eff}}^{(2)}(M)$ is the definition of the general ADS superpotential $\mathcal{W}_{\text{ADS}}^{N_F, N}$.

II Monopoles in SYM on $\mathbb{R}^3 \times S^1$

In this section we consider the pure $\mathcal{N} = 1$ supersymmetric Yang-Mills theory coupled to a background chiral superfield $T^{(\mu)}$

$$\mathcal{Z}(T^{(\mu)}) = \int \mathcal{D}W \, e^{i \int d^4x \mathcal{L}(W, T^{(\mu)})} = e^{-i \left(\int d^4x d^2\theta \, \mathcal{W}_{\text{eff}}(T^{(\mu)}) + \text{h.c.} \right)} . \quad (2.1)$$

Here W denotes the $SU(N)$ field-strength chiral superfield, and $T^{(\mu)}$ is a non-dynamical background chiral superfield

$$T^{(\mu)}(y, \theta) \equiv \tau(y) + \sqrt{2} \theta^\alpha \chi_\alpha^\tau(y) + \theta^2 F^\tau(y) , \quad \langle \tau \rangle = 4\pi i g^2(\mu) + \theta(\mu) 2\pi \quad (2.2)$$

The superscript μ indicates the (high) energy scale where $T^{(\mu)}$ becomes dynamical. The microscopic lagrangian defining the theory reads

$$\mathcal{L} = 14\pi \, \text{Im} \, \text{tr}_N \int d^2\theta \, T^{(\mu)} W^\alpha W_\alpha . \quad (2.3)$$

The dynamical quantities we are after are the effective superpotential $\mathcal{W}_{\text{eff}}(T^{(\mu)})$ defined in Eq. (2.1) and the gaugino condensate $\langle \text{tr}_N \lambda^2 \rangle$ in the presence of the τ field. The two quantities are related due to a functional identity

$$\langle \text{tr}_N \lambda^2 \rangle = 8\pi \mathcal{Z}(T^{(\mu)}) \delta \delta F^\tau(x) \mathcal{Z}(T^{(\mu)}) \Big|_{T^{(\mu)}(y, \theta) = \tau} = -8\pi i \, \partial \mathcal{W}_{\text{eff}}(\tau) \partial \tau , \quad (2.4)$$

which trivially follows from Eqs. (2.1)-(2.3).

Gauge Theory on $\mathbb{R}^3 \times S^1$

Let x_4 be periodic with period $\beta/2\pi$.¹ We must then impose periodic boundary conditions

¹The indices run over $m = 1, 2, 3, 4$ and $\mu = 1, 2, 3$. Our conventions are the same as in Ref. [2].

for bosons, $A_m(x_\mu, x_4 = 0) = A_m(x_\mu, x_4 = \beta)$ and fermions, $\lambda(x_\mu, x_4 = 0) = \lambda(x_\mu, x_4 = \beta)$ to preserve supersymmetry. In addition, the local gauge group itself must also be composed of gauge transformations periodic on S^1 : $U(x_\mu, x_4 = 0) = U(x_\mu, x_4 = \beta)$. We now summarize the results of the analysis carried out in Ref. [2].

1. There are N classically flat directions of the vacuum moduli space of the compactified theory parametrized by $\langle A_4 \rangle = \text{diag}(a_1, a_2, \dots, a_N)$. This classical moduli space is lifted non-trivially at the quantum level to

$$\langle A_4 \rangle = \text{diag}\left(N - 1N\pi i\beta, N - 3N\pi i\beta, \dots, -N - 1N\pi i\beta\right), \quad (2.5)$$

by the monopole-generated superpotential derived in the section III of [2]. The distinctive feature of (2.5) is the constant equal spacing between the VEVs a_j .

2. The semi-classical physics of the $\mathbb{R}^3 \times S^1$ $SU(N)$ theory is described by configurations of monopoles of N different types. In general, the field configurations of the $\mathbb{R}^3 \times S^1$ theory which are relevant in the semi-classical regime are both instantons and monopoles. Remarkably, the instantons on $\mathbb{R}^3 \times S^1$ can be understood as composite configurations of N single monopoles, one of each of the N different types [22–24]. One expects in an $SU(N)$ theory on \mathbb{R}^4 that the lowest charged monopoles come in $N - 1$ different varieties, carrying a unit of magnetic charge from each of the $U(1)$ factors of the $U(1)^{N-1}$ gauge group left unbroken by the VEVs. The additional monopole, needed to make up the N types, is specific to the compactification on $\mathbb{R}^3 \times S^1$ [23, 24]. The magnetic charge of the new monopole is such that when all N types of monopoles are present, the magnetic charges cancel and the resulting configuration only carries a unit instanton charge.

Monopole calculus in $SU(2)$

The standard BPS monopole solution in Hedgehog gauge [25] is

$$A_4^{\text{BPS}}(x_\nu) = (v|x| \coth(v|x|) - 1)x_a|x|^2\tau^a 2i, \quad A_\mu^{\text{BPS}}(x_\nu) = \left(1 - v|x|\sinh(v|x|)\right)\epsilon_{\mu\nu a}x_\nu|x|^2\tau^a 2i. \quad (2.6)$$

These expressions are obviously independent of the S^1 variable x_4 , since the latter can be thought of as the time coordinate of the static monopole. The boundary values of (2.6) as $|x| \rightarrow \infty$, when gauge rotated to unitary (singular) gauge, agree with (2.5) for $v = \pi/\beta$

$$A_4^{\text{BPS}} \rightarrow v\tau^3 2i = \pi\beta\tau^3 2i = \langle A_4 \rangle. \quad (2.7)$$

The monopole solution (2.6) satisfies the self-duality equations and has topological charge

$$Q \equiv 116\pi^2 \int_0^\beta dx_4 \int d^3x \text{tr}^* F_{mn} F^{mn} = \beta v 2\pi = 12. \quad (2.8)$$

The monopole has magnetic charge one, instanton charge zero, and the action S_{BPS} is

$$S_{\text{BPS}} = 4\pi g^2(\mu)\beta v = 4\pi^2 g^2(\mu) , \quad S_{\text{BPS}}(T^{(\mu)}) = -i\pi T^{(\mu)} . \quad (2.9)$$

The solution (2.6) precisely two adjoint fermion zero modes $\lambda_\alpha^{\text{BPS}} = 12\xi_\beta(\sigma^m \bar{\sigma}^n)_\alpha^\beta F_{mn}^{\text{BPS}}$, with normalization [26]:

$$\int d^3 X \int d^2 \xi \text{tr} (\lambda^{\text{BPS}}{}^\alpha(x) \lambda_\alpha^{\text{BPS}}(x)) = 2g^2 S_{\text{BPS}} \beta = 8\pi^2 \beta . \quad (2.10)$$

Here σ^m and $\bar{\sigma}^n$ are the four Pauli matrices and ξ_β is the two-component Grassmann collective coordinate. The semi-classical integration measure of the standard single-monopole on $\mathbb{R}^3 \times S^1$ reads [26]:

$$\int d\mu^{\text{BPS}} = \mu^3 e^{-S_{\text{BPS}}(T^{(\mu)})} \int d^3 X (2\pi)^{3/2} 8\pi^3 \int_0^{2\pi} d\Omega \sqrt{2\pi} 2\pi v \int d^2 \xi 18\pi^2 . \quad (2.11)$$

This measure is obtained in the standard way by changing variables in the path integral from field-fluctuations around the monopole to the monopole's collective coordinates. The UV-regularized determinants over non-zero eigenvalues of the quadratic fluctuation operators cancel between fermions and bosons due to supersymmetry. The ultra-violet divergences are regularized in the Pauli-Villars scheme, which explains the appearance of the Pauli-Villars mass scale μ . We can now compute the single monopole contribution to $\langle \text{tr} \lambda^2 \rangle$ as in Ref. [2]:

$$\langle \text{tr} \lambda^2 \rangle_{\text{BPS}} = \int d\mu^{\text{BPS}} \text{tr} (\lambda^{\text{BPS}}{}^\alpha(x) \lambda_\alpha^{\text{BPS}}(x)) = 8\pi^2 \mu^3 \exp[i\pi T^{(\mu)}] . \quad (2.12)$$

As explained in Ref. [2] the monopole of the second type will give a contribution identical to (2.12). Putting the two together we get

$$\langle \text{tr} \lambda^2 \rangle = 16\pi^2 \mu^3 \exp[i\pi T^{(\mu)}] . \quad (2.13)$$

Substituting (2.13) into (2.4) gives the equation for the superpotential $\mathcal{W}_{\text{eff}}(\tau)$ with the solution

$$\mathcal{W}_{\text{eff}}(T^{(\mu)}) = 2\mu^3 \exp[\pi i T^{(\mu)}] . \quad (2.14)$$

The expressions (2.13) and (2.14) are, in spite of appearances, renormalization group invariant and μ -independent. We also note that the dependence on the S^1 radius β disappeared, and the decompactification limit $\beta \rightarrow \infty$ does not change the final results (2.13) and (2.14).

Generalization to SU(N)

The calculation of the gaugino condensate and the superpotential can be straightforwardly generalized to the case of gauge group $SU(N)$. The quantum vacuum has

$$a_j - a_{j+1} = 2\pi i N \beta \bmod 2\pi i \beta, \quad j = 1, 2, \dots, N, \quad (2.15)$$

and each of the N types of monopoles has equal actions and equal topological charges:

$$S_{\text{mono}} = 8\pi^2 N g^2(\mu), \quad S_{\text{mono}}(T^{(\mu)}) = -2\pi i N T^{(\mu)}, \quad Q_{\text{mono}} = 1N. \quad (2.16)$$

The contribution of N monopoles to the gaugino condensate and the superpotential is straightforward:

$$\langle \text{tr} \lambda^2 \rangle = 16\pi^2 \mu^3 \exp[2\pi i N T^{(\mu)}], \quad \mathcal{W}_{\text{eff}}(T^{(\mu)}) = N \mu^3 \exp[2\pi i N T^{(\mu)}]. \quad (2.17)$$

This concludes our analysis of supersymmetric QCD with $N_F = 0$.

III ADS Superpotential in Supersymmetric QCD

The microscopic theory is defined in terms of the $SU(N)$ vector superfield coupled to N_F chiral superfields in the $[N]$ representation, Q_{ur} ($u = 1, \dots, N$; $r = 1, \dots, N_F$), and to N_F chiral superfields in the $[\bar{N}]$ representation, \tilde{Q}^{ru} . The global classical symmetry of the lagrangian is

$$SU(N_F)_{\text{left}} \times SU(N_F)_{\text{right}} \times U(1)_V \times U(1)_A \times U(1)_R, \quad (3.1)$$

where $U(1)_R$ is the anomaly-free combination of the R-symmetry and the axial $U(1)_A$:

$$W(\theta) \rightarrow e^{-i\alpha} W(e^{i\alpha}\theta), \quad Q(\theta) \rightarrow e^{i\alpha(N-N_F)/N_F} Q(e^{i\alpha}\theta), \quad \tilde{Q}(\theta) \rightarrow e^{i\alpha(N-N_F)/N_F} \tilde{Q}(e^{i\alpha}\theta). \quad (3.2)$$

At the quantum level the classical $U(1)_A$ symmetry is anomalous, and the global *quantum* symmetry of the lagrangian is $SU(N_F)_{\text{left}} \times SU(N_F)_{\text{right}} \times U(1)_V \times U(1)_R$.

The classical vacuum state is determined in the standard way via the D-flatness condition and can be brought to a simple ‘rectangular diagonal’ form:

$$\langle A_{ur} \rangle = \left\{ \delta_{ur} v_r, \quad u=1, \dots, N_F, \quad u=N_F+1, \dots, N \right\} = \langle \tilde{A}_{ur}^\dagger \rangle. \quad (3.3)$$

The N_F complex vacuum expectation values v_1, \dots, v_{N_F} are not fixed by the classical lagrangian, and parameterize the N_F -complex-dimensional classical vacuum moduli space of the theory. It is now straightforward to determine which symmetries of the lagrangian are

left unbroken by the classical vacuum (3.3). The $SU(N)$ gauge symmetry is spontaneously broken by (3.3) to $SU(N - N_F)$. In order to determine the surviving global symmetry it is convenient to restrict ourselves to the case of all VEVs equal, $v_1 = v_2 = \dots \equiv v$. This choice will not affect the counting of classically massless degrees of freedom, but will simplify the symmetry reasoning. This vacuum state breaks the $U(1)_A$ and $U(1)_R$. It also breaks $SU(N_F)_{\text{left}} \times SU(N_F)_{\text{right}}$ to the diagonal $SU(N_F)$ subgroup that rotates A_r and \tilde{A}_r^\dagger in the same way and is further compensated by the gauge transformation, such that the vacuum is unchanged. Thus, the global symmetry respected by the vacuum is $SU(N_F) \times U(1)_V$. Hence, we expect $N_F^2 + 1$ massless Goldstone bosons at the perturbative level, i.e. $N_F^2 + 1$ real massless scalars coming from the broken generators of the global $SU(N_F) \times U(1)_A \times U(1)_R$. When the non-perturbative effects are taken into account one of these massless degrees of freedom will acquire a mass due to the $U(1)_A$ anomaly, and N_F^2 real scalar degrees of freedom will remain massless.

At the same time, the number of *classically massless* real scalar degrees of freedom is $2N_F^2$. This is twice the number of exact Goldstone bosons. The remaining N_F^2 classically massless real scalars must be lifted by a non-perturbatively generated superpotential. The functional form of this superpotential Eq. (1.1) was uniquely determined in [1].

The ADS superpotential appears in the low-energy effective description of the microscopic theory with $N_F \leq N - 1$. It is important to distinguish between the two cases: $N_F = N - 1$ where the gauge group is completely broken by the vacuum; and $N_F < N - 1$ where the non-Abelian gauge subgroup $SU(N - N_F)$ is unbroken. The first case is relatively simple and is well understood [18, 17, 1]. For $N_F = N - 1$ the superpotential is generated at the 1-instanton level, and since the gauge group is completely broken, the instanton calculation is reliable and infra-red safe. In the second, more general case, $N_F < N - 1$, instantons are known to give trivially vanishing contributions to (1.1), nevertheless the renormalization group decoupling argument from $N_F = N - 1$ to $N_F < N - 1$ requires the superpotential to be non-vanishing, and determines the normalization constant in (1.1) [21, 18, 17] to be $N - N_F$.

$N_F < N - 1$: Step One

For $N_F < N - 1$ there is an unbroken gauge subgroup $SU(N - N_F)$ and as discussed in the Introduction, it is important to realise that the generation of the ADS superpotential is the two-step process represented by Eq. (1.3). We first look at the perturbative decoupling of the massive vector bosons and Higgs bosons. This step is the generalization of the Affleck-Dine-Seiberg $N_F = 1$ argument from Section 4 of Ref. [1] to all $N_F < N - 1$. The relevant matter degrees of freedom are the $2N_F^2$ classically massless real scalars and

their superpartners. They can be packaged into N_F^2 chiral superfield (complex) degrees of freedom. In fact, there are precisely N_F^2 gauge-invariant composite chiral meson superfields $M_s^r(x, \theta) = \tilde{Q}^{ru}(x, \theta)Q_{us}(x, \theta)$. As the scalar components of the superfields M have the mass-dimension two we prefer to use an equivalent parametrisation of matter degrees of freedom in terms of mass-dimension one chiral superfields

$$\Phi_s^r = v\sqrt{2} \log \tilde{Q}^r Q_s v^2 . \quad (3.4)$$

The logarithm is used for the later convenience and the mass-dimension one in (3.4) is achieved via explicit dependence on the classical VEV v which will be necessary in the perturbative decoupling treatment, but will disappear from the final results. In fact, all the non-renormalizable interactions induced by integrating out the heavy fields will be in terms of higher dimensional operators of the light fields divided by the powers of v . A perturbative (or semi-classical) treatment is applicable as long as v is large compared to $\Lambda_{N_F, N}$, which will be assumed. There is a unique operator of dimension five which couples the two sectors,

$$\Gamma = \sqrt{2}32\pi^2 v \int d^2\theta (\text{Tr}_{N_F} \Phi) W^{a\alpha} W_\alpha^a + \text{h.c.} . \quad (3.5)$$

The the normalization factor $\sqrt{2}32\pi^2$ can be determined either from the corresponding 1-loop perturbative supergraphs, or from requiring the $U(1)_R$ invariance of the effective lagrangian

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{(1)} = & \sum_{a=1}^{(N-N_F)^2-1} \int d^2\theta \left(14g^2(\mu) W^{a\alpha} W_\alpha^a + \sqrt{2}32\pi^2 v (\text{Tr}_{N_F} \Phi) W^{a\alpha} W_\alpha^a \right) + \text{h.c.} \\ & + \int d^2\theta d^2\bar{\theta} \Phi^\dagger \Phi . \end{aligned} \quad (3.6)$$

Under $U(1)_R$ the pure gauge term is anomalous

$$\delta 14g^2 \int d^2\theta W^2 = -(N - N_F)16\pi^2 F^{mn*} F_{mn} , \quad (3.7)$$

and the $U(1)_R$ transformation of Φ , read from (3.2),

$$\delta \Phi_s^r = iv\sqrt{2}(N - N_F)N_F \quad (3.8)$$

leaves (3.6) invariant as required.

There are of course even higher-dimensional operators coupling the two sectors, but they are suppressed by higher powers of v and will not contribute at the relevant order. Furthermore, perturbative non-renormalization theorems will prevent the generation of the superpotential made solely of the Φ fields as no such superpotential existed classically. Finally, note that Eq. (3.6) should be thought of as the Wilson effective lagrangian with μ in $g^2(\mu)$ on the

right hand side of (3.6) being the Wilson scale. In deriving $\mathcal{L}_{\text{eff}}^{(1)}$ we have integrated out all the degrees of freedom with masses and virtualities greater than μ , where $\mu \leq v$.

$N_F < N - 1$: Step Two

The gauge sector together with the gauge-meson interactions sector of the effective lagrangian $\mathcal{L}_{\text{eff}}^{(1)}$ can be represented as

$$\mathcal{L}_{\text{eff}}^{\text{gauge}} = 18\pi \text{Im} \sum_{a=1}^{(N-N_F)^2-1} \int d^2\theta T^{(\mu)} W^{a\alpha} W_\alpha^a \quad (3.9)$$

where² we have introduced a chiral superfield

$$T^{(\mu)} = 4\pi i g^2(\mu) + 12\pi i \sqrt{2} \text{Tr}_{N_F} \Phi v. \quad (3.10)$$

We now see from equations (1.3) and (3.9) that the problem of deriving the ADS superpotential in the theory with N colours and N_F flavours is reduced to integrating out the vector supermultiplet $W_{SU(N-N_F)}$ in the pure $SU(N-N_F)$ supersymmetric Yang-Mills theory coupled to the background superfield $T^{(\mu)}$

$$\int \mathcal{D}W_{SU(N-N_F)} e^{i \int d^4x \mathcal{L}_{\text{eff}}^{\text{gauge}}(W_{SU(N-N_F)}, T^{(\mu)})} = e^{-i(\int d^4x d^2\theta \mathcal{W}_{\text{eff}}(T^{(\mu)}) + \text{h.c.})}. \quad (3.11)$$

Applying the analysis of section II we conclude that the functional integral over the gauge supermultiplet receives contributions from $N - N_F$ varieties of monopoles which arise in the $SU(N - N_F)$ supersymmetric Yang-Mills theory partially compactified on $\mathbb{R}^3 \times S^1$. The semi-classical evaluation of the integral in the monopole background is exact and in the decompactification limit $\mathbb{R}^3 \times S^1 \rightarrow \mathbb{R}^4$ gives, in analogy with Eq. (2.17)

$$\mathcal{W}_{\text{eff}}(T^{(\mu)}) = (N - N_F) \mu^3 \exp \left[2\pi i N - N_F T^{(\mu)} \right]. \quad (3.12)$$

In the rest of this section we want to demonstrate that the superpotential $\mathcal{W}_{\text{eff}}(T^{(\mu)})$, given by Eq. (3.12) is in fact equal to the ADS superpotential \mathcal{W}_{ADS} (1.1). Indeed, substitute the definition of $T^{(\mu)}$, Eq. (3.10) into Eq. (3.12),

$$\mathcal{W}_{\text{eff}}(T^{(\mu)}) = (N - N_F) \mu^3 e^{-8\pi^2 g^2(\mu) (N - N_F)} \left(v^{2N_F} \det_{N_F}(\tilde{Q}^r Q_s) \right)^{1N - N_F}. \quad (3.13)$$

Now note that the μ -dependent terms combine to the RG invariant definition of the dynamical scale of the supersymmetric pure $SU(N - N_F)$ gauge theory

$$\mu^3 e^{-8\pi^2 g^2(\mu) (N - N_F)} = \Lambda_{0, N - N_F}^3. \quad (3.14)$$

² $\mathcal{L}_{\text{eff}}^{\text{gauge}}$ is defined as the first line on the right hand side of Eq. (3.6), i.e. $\mathcal{L}_{\text{eff}}^{(1)}$ minus the pure matter sector.

Furthermore, the decoupling relation, $\Lambda_{N_F, N}^{3N-N_F} = v^{2N_F} \Lambda_{0, N-N_F}^{3(N-N_F)}$ enables us to rewrite the final result in terms of the dynamical scale $\Lambda_{N_F, N}$ of the original SQCD:

$$\mathcal{W}_{\text{eff}} = (N - N_F) \left(\Lambda_{N_F, N}^{3N-N_F} \det_{N_F}(\tilde{Q}^r Q_s) \right)^{1N-N_F} = \mathcal{W}_{\text{ADS}}^{N_F, N}. \quad (3.15)$$

Q.E.D.

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